MA 126 — Fall 2016 — Prof. Clontz — Practice Test

Name: Solutions

- Each question is labeled with its worth toward the grade for this exam.
- **Choose SEVEN of the eight non-bonus problems to solve. Mark the one you don't want graded by marking it with the word "SKIP" in the upperleft corner of the page.** If you fail to do so, you will be penalized 15 points.
- Show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- You may use at most two pages (front and back) of 8.5×11 inch paper of notes in your own handwriting.
- You may borrow a calculator from the instructor. None of the questions require the use of a calculator.
- This exam is due after 50 minutes. Tests submitted over one minute late will not be graded.
- Your score for this practice test will be divided by 5 and rounded up to the nearest integer, and will go toward your Quiz grade for the semester. A maximum of $110/5 = 22$ points are possible.

1. (15 points) Show that $\int \frac{2x-5}{x^2}\,dx = 2\ln|x| + \frac{5}{x} + C$.

$$\int \frac{2x-5}{x^2}\,dx = \int \frac{2x}{x^2} - \frac{5}{x^2}\,dx$$

$$= \int 2\frac{1}{x} - 5x^{-2}\,dx$$

$$= 2\ln|x| - 5(-x^{-1}) + C$$

$$= \boxed{2\ln|x| + 5/x + C}$$

2. (15 points) Use these definitions:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}.$$

to prove the following identity:

$$\sinh^2 x = \frac{1}{2}\cosh 2x - \frac{1}{2}.$$

$$\sinh^2 x = \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x} - 2e^{\cancel{x}}\cancel{e^{-x}} + e^{-2x}}{4}$$

$$= \frac{e^{2x} - 2 + e^{-2x}}{4}$$

$$\frac{1}{2}\cos 2x - \frac{1}{2} = \frac{1}{2}\,\frac{e^{2x} + e^{-2x}}{2} - \frac{1}{2}$$

$$= \frac{e^{2x} + e^{-2x}}{4} - \frac{2}{4}$$

$$= \frac{e^{2x} - 2 + e^{-2x}}{4} \qquad \square$$

3. (15 points) Prove that the slope of the line tangent to the curve $y = \cosh(2x)$ at the point $(\ln 2, \frac{17}{8})$ is $\frac{15}{4}$. (Hint: the slope of the line tangent to the curve $y = f(x)$ at $(x_0, f(x_0))$ is $f'(x_0)$.)

$$f(x) = \cosh(2x)$$

$$f'(x) = 2\sinh(2x)$$

$$f'(\ln 2) = 2\sinh(2\ln 2)$$

$$= 2\sinh(\ln 4)$$

$$= \cancel{2}\,\frac{\cancel{e}^{\ln 4} - \cancel{e}^{-\ln 4}}{\cancel{2}}$$

$$= 4 - \frac{1}{4}$$

$$= 15/4 \quad \square$$

4. (15 points) Find $\int 9x^2 \sinh(x^3+4)\,dx$. (Hint: use a direct substitution.)

Let $u = x^3 + 4$

$du = 3x^2\,dx$

$3\,du = 9x^2\,dx$

$$= \int 3\sinh(u)\,du$$

$$= 3\cosh(u) + C$$

$$= \boxed{3\cosh(x^3+4) + C}$$

5. (15 points) Show that $\int_4^5 6r\sqrt{r^2-16}\,dr = 54$. (Hint: use a direct substitution.)

$r=5 \to u = 25-16 = 9$
$r=4 \to u = 16-16 = 0$

Let $u = r^2 - 16$

$du = 2r\,dr$

$3\,du = 6r\,dr$

$$\int_{u=0}^{u=9} 3(u)^{1/2}\,du$$

$$= \cancel{3}\left[\frac{2}{\cancel{3}} u^{3/2}\right]_0^9$$

$$= 2(9)^{3/2} - \cancel{2(0)^{3/2}}$$

$$= 2(3)^3$$

$$= 2(27) = \boxed{54}$$

6. (15 points) Find $\int 3y \cosh(y)\, dy$. (Hint: use integration by parts.)

Let $u = 3y$ $\quad v = \sinh(y)$

$du = 3dy$ $\quad dv = \cosh(y)dy$

$$= 3y\sinh(y) - \int 3\sinh(y)dy$$

$$= \boxed{3y\sinh(y) - 3\cosh(y) + C}$$

7. (15 points) Use integration by parts with cycling to show

$$\int 2e^x \cos(x)\,dx = e^x \sin(x) + e^x \cos(x) + C.$$

$$\int 2e^x \cos(x)\,dx = 2e^x \sin(x) - \int 2e^x \sin(x)\,dx$$

Let $u = 2e^x$ $v = \sin(x)$
$du = 2e^x dx$ $dv = \cos(x)dx$

Let $u = 2e^x$ $v = -\cos x$
$du = 2e^x dx$ $dv = \sin x\,dx$

$$= 2e^x \sin(x) - \left[-2e^x \cos(x) - \int -2e^x \cos(x)dx\right]$$

$$\int 2e^x \cos(x)dx = 2e^x \sin(x) + 2e^x \cos(x) - \cancel{\int 2e^x \cos(x)dx}$$

$$+ \cancel{\int 2e^x \cos(x)dx}$$

$$+ \int 2e^x \cos(x)dx$$

$$2\int 2e^x \cos(x)dx = 2e^x \sin(x) + 2e^x \cos(x) + C$$

$$\int 2e^x \cos(x)dx = e^x \sin(x) + e^x \cos(x) + C \quad \square$$

8. (15 points) Find $\int \sin^3 x \cos^3 x \, dx$. (Hint: use the methods of trigonometric integration.)

$$= \int \sin^3 x \cos^2 x \cos x \, dx$$

$$= \int \sin^3 x (1 - \sin^2 x) \cos x \, dx$$

Let $u = \sin x$

$du = \cos x \, dx$

$$= \int u^3 (1 - u^2) \, du$$

$$= \int u^3 - u^5 \, du$$

$$= \frac{1}{4} u^4 - \frac{1}{6} u^6 + C$$

$$= \boxed{\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C}$$

OR

$$= \int \cos^3 x (1 - \cos^2 x) \sin x \, dx$$

Let $u = \cos x$

$du = -\sin x \, dx$

$$= \int u^3 (1 - u^2)(-du)$$

$$= \int u^5 - u^3 \, du$$

$$= \boxed{\frac{1}{6} \cos^6 x - \frac{1}{4} \cos^4 x + C}$$

9. (BONUS, 5 points) Use the following formula

$$\int x^{n+1}e^x\,dx = x^{n+1}e^x - (n+1)\int x^n e^x\,dx$$

to find

$$\int x^5 e^x\,dx.$$

$= x^5e^x - 5\int x^4e^x\,dx$

$= x^5e^x - 5\left[x^4e^x - 4\int x^3e^x\,dx\right]$

$= x^5e^x - 5x^4e^x + 20\left[x^3e^x - 3\int x^2e^x\,dx\right]$

$= x^5e^x - 5x^4e^x + 20x^3e^x - 60\left[x^2e^x - 2\int xe^x\,dx\right]$

$= x^5e^x - 5x^4e^x + 20x^3e^x - 60x^2e^x + 120\left[xe^x - \int \cancel{x} e^x\,dx\right]$

$= \boxed{x^5e^x - 5x^4e^x + 20x^3e^x - 60x^2e^x + 120xe^x - 120e^x + C}$