(1) Find $\int 6x^2 \sinh(x^3)\,dx$.

Let $u = x^3$

$du = 3x^2\,dx$

$2\,du = 6x^2\,dx$

$= \int 2 \sinh(u)\,du$

$= 2\cosh(u) + C$

$= \boxed{2\cosh(x^3) + C}$

② Find $\int e^{2\theta} \cos\theta \, d\theta$.

Let $u = e^{2\theta}$ $\quad v = \sin\theta$

$du = 2e^{2\theta} d\theta$ $\quad dv = \cos\theta \, d\theta$

$$= e^{2\theta} \sin\theta - 2\int e^{2\theta} \sin\theta \, d\theta$$

Let $u = e^{2\theta}$ $\quad v = -\cos\theta$

$du = 2e^{2\theta} d\theta$ $\quad dv = \sin\theta \, d\theta$

$$= e^{2\theta} \sin\theta - 2\left[-e^{2\theta}\cos\theta + 2\int e^{2\theta}\cos\theta \, d\theta\right]$$

$$\int e^{2\theta}\cos\theta \, d\theta = e^{2\theta}\sin\theta + 2e^{2\theta}\cos\theta - \cancel{4\int e^{2\theta}\cos\theta \, d\theta}$$

$$+4\int \sim d\theta \qquad\qquad +4\int \sim d\theta$$

$$5\int \sim d\theta = e^{2\theta}\sin\theta + 2e^{2\theta}\cos\theta + C$$

$$\int \sim d\theta = \boxed{\frac{1}{5}e^{2\theta}\sin\theta + \frac{2}{5}e^{2\theta}\cos\theta + C}$$

③ Find $\int \frac{2}{4+z^2}\,dz$.

Let $4+z^2 = 4+4\tan^2\theta = 4\sec^2\theta$

$z = 2\tan\theta \longrightarrow \theta = \tan^{-1}(z/2)$

$dz = 2\sec^2\theta\, d\theta$

$= \int \frac{\cancel{2}}{\cancel{4\sec^2\theta}} \cancel{2\sec^2\theta}\, d\theta$

$= \int d\theta$

$= \theta + C$

$= \boxed{\tan^{-1}(z/2) + C}$

(4) Find $\int \frac{6y^2+2}{y^3+y}\,dy$.

$$\frac{6y^2+2}{y(y^2+1)} = \frac{A}{y} + \frac{By+C}{y^2+1}$$

$$6y^2+2 = A(y^2+1) + (By+C)(y)$$

Let $y=0$

$$2 = A(0+1) + 0$$

$$2 = A$$

$$6y^2+\cancel{2} = 2y^2+\cancel{2} + (By+C)(y)$$

$$-2y^2 \quad -2y^2$$

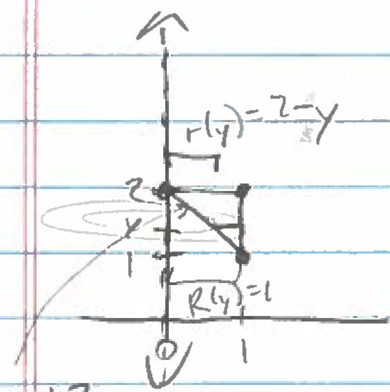
$$4y^2+0y = By^2 + Cy$$

y^2: $4 = B$

y: $0 = C$

$$= \boxed{\int\left(\frac{2}{y} + \frac{4y}{y^2+1}\right)dy = 2\ln|y| + 2\ln(y^2+1) + C}$$

(5) Use the washer method to find the volume of the solid of revolution obtained by rotating the triangle with vertices (1,1), (1,2), (0,2) around the y-axis.



$y = -x + 2$

$x = 2 - y$

$$\boxed{V = \pi \int_1^2 \left((1)^2 - (2-y)^2\right) dy}$$

$$= \pi \int_1^2 (1 - 4 + 4y - y^2)\, dy$$

$$= \pi \int_1^2 (-y^2 + 4y - 3)\, dy$$

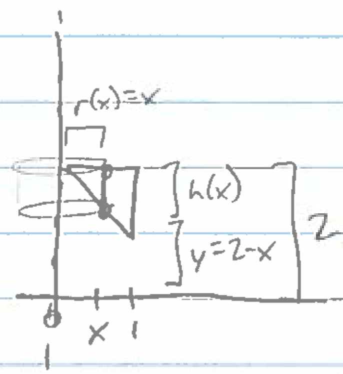
$$= \pi \left[-\frac{1}{3}y^3 + 2y^2 - 3y \right]_1^2$$

$$= \pi \left[\left(-\frac{8}{3} + 8 - 6\right) - \left(-\frac{1}{3} + 2 - 3\right) \right]$$

$$= \pi \left[-\frac{2}{3} - \left(-\frac{4}{3}\right) \right]$$

$$= \boxed{\frac{2\pi}{3}}$$

6) Use cylindrical shell... (see #5)



$h(x) + 2 - x = 2$

$h(x) = x$

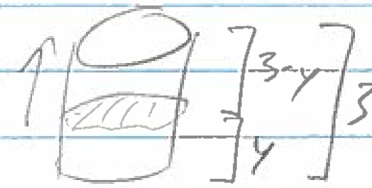
$$\boxed{V = 2\pi \int_0^1 (x)(x)\,dx}$$

$$= 2\pi \int_0^1 x^2\,dx$$

$$= 2\pi \left[\frac{1}{3}x^3\right]_0^1$$

$$= \boxed{2\pi/3}$$

7) A cylindrical tank of height 3 meters and diameter 4 meters has a cross-sectional area of 4π square meters at any height. Find the work required to empty the tank (assume $\rho = 10{,}000$).



$$A = 4\pi$$

$$dV = 4\pi\, dy$$

$$dF = 40000\pi\, dy$$

$$dW = 40000\pi(3-y)\,dy$$

$$\boxed{W = \int_0^3 40{,}000\pi(3-y)\,dy}$$

$$= 40{,}000\pi\left[3y - \tfrac{1}{2}y^2\right]_0^3$$

$$= 40{,}000\pi\left[9 - \tfrac{9}{2}\right]$$

$$= \frac{360{,}000}{2}\pi$$

$$\boxed{= 180{,}000\pi}$$