(8) What Maclaurin Series should be used to approximate $e^{-1/2} = \frac{1}{\sqrt{e}}$?

$$e^x = \boxed{\sum_{k=0}^{\infty} \frac{x^k}{k!}} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

(9) Find the error term $R_n(x)$ for $f(x) = e^x$, where x_n is between 0 and x.

$$R_n(x) = \frac{f^{(n+1)}(x_n)}{(n+1)!}(x-0)^{n+1}$$

$$= \frac{e^{x_n}}{(n+1)!} x^{n+1}$$

(10) Approximate $e^{-1/2} = \frac{1}{\sqrt{e}}$ with an error no greater than $\frac{1}{1000}$.

$$\left|R_n(-\tfrac{1}{2})\right| = \frac{e^{x_n}}{(n+1)!}\left|-\tfrac{1}{2}\right|^{n+1} \quad \text{where} \quad -\tfrac{1}{2} \le x_n \le 0$$

$$= \frac{e^{x_n}}{(n+1)!\,2^{n+1}}$$

$$\le \frac{\cancel{e}^{0}}{(n+1)!\,2^{n+1}} = \frac{1}{(n+1)!\,2^{n+1}}$$

$$\left|R_4(-\tfrac{1}{2})\right| \le \frac{1}{5!\;2^5} = \frac{1}{(120)(32)} < \frac{1}{3200} < \frac{1}{1000}$$

$$\left(\left|R_3(-\tfrac{1}{2})\right| \le \frac{1}{4!\,2^4} = \frac{1}{(24)(16)} = \frac{1}{384} \text{ is too big.}\right)$$

So at $x = -\frac{1}{2}$,

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$$

24×16

$$\frac{1}{\sqrt{e}} = e^{-1/2} \approx 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384}$$

$$\approx 1 - 0.5 + 0.125 - 0.0208 + 0.0026$$

$$\approx \boxed{0.6068}$$

(Scientific Calculator gives

0.60653...)