(5) Find a power series converging to $\frac{x^3}{e^{x^2}}$.

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{-x^2} = \frac{1}{e^{x^2}} = \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!} = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{k!}$$

$$\frac{x^3}{e^{x^2}} = x^3 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k}$$

$$= \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k+3}} \quad \left(= x^3 - x^4 + \frac{x^5}{2} - \frac{x^6}{6} + \cdots \right)$$

⑥ Find a power series converging to $\frac{1}{x^2+2x+1}$ for $|x|<1$.

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$(1+x)^{-1} = \frac{1}{1+x} = \sum_{k=0}^{\infty} (-x)^k = \sum_{k=0}^{\infty} (-1)^k x^k$$

$$\frac{d}{dx}\left[(1+x)^{-1}\right] = -(1+x)^{-2} = \frac{d}{dx}\sum_{k=0}^{\infty} (-1)^k x^k = \sum_{k=0}^{\infty} (-1)^k k x^{k-1}$$

$$\frac{+1}{(1+x)^2} = -\sum_{k=0}^{\infty} (-1)^k k x^{k-1}$$

$$\frac{1}{x^2+2x+1} = \boxed{\sum_{k=0}^{\infty} (-1)^{k+1} k x^{k-1}} \quad \left(= 0 - 1 + 2x - 3x^2 + \dots\right)$$

(7) Find a power series converging to $\ln|x|$ for $0<x<2$.

$$\frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k \qquad (\text{for } |x|<1)$$

$$\ln|1+x| = \int \sum_{k=0}^{\infty} (-1)^k x^k \qquad (\text{for } |x|<1)$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1} \cancel{+C}$$

Let $x=0$

$$\cancel{\ln|1+0|} = \cancel{\sum_{k=0}^{\infty} (-1)^k \frac{0}{k+1}} + C$$

$$0 = 0 + C$$

$$C=0$$

$$\ln|x| = \ln|1+(x-1)| = \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{(x-1)^{k+1}}{k+1}} \quad \left(\begin{array}{l}\text{for } |x-1|<1 \\ -1<x-1<1 \\ 0<x<2\end{array}\right)$$

$$\left(= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots \right)$$

⑧ Generate the Maclaurin Series for $\cosh x$.

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$f^{(0)}(x) = \cosh x \quad \rightarrow \quad f^{(2k)}(0) = 1$$

$$f^{(1)}(x) = \sinh x \quad \rightarrow \quad f^{(2k+1)}(0) = 0$$

$$f^{(2)}(x) = \cosh x$$

$$= \sum_{k=0}^{\infty} \left(\frac{\cancel{f^{(2k)}(0)}}{(2k)!} x^{2k} + \cancel{\frac{f^{(2k+1)}(0)}{(2k+1)!} x^{2k+1}} \right)$$

$$= \boxed{\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}}$$

(9) Find a power series converging to $x\cos x$.

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$x\cos x = x\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$= \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k)!}} \quad \left(= x - \frac{x^3}{2} + \frac{x^5}{24} - \frac{x^7}{720} + \cdots \right)$$

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(10) Find a power series converging to $\frac{d}{dx}\left[\sin(x^2)\right]$

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\sin(x^2) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+2}}{(2k+1)!}$$

$$\frac{d}{dx}\left[\sin(x^2)\right] = \sum_{k=0}^{\infty} (-1)^k \frac{\frac{d}{dx}\left[x^{4k+2}\right]}{(2k+1)!}$$

$$= \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{(4k+2)\,x^{4k+1}}{(2k+1)!}}$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{2\cancel{(2k+1)}\,x^{4k+1}}{(2k)!\cancel{(2k+1)}}$$

OR

$$\frac{d}{dx}\left[\sin(x^2)\right] = 2x\cos(x^2)$$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\cos(x^2) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{4k}}{(2k)!}$$

$$2x\cos(x^2) = \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{2x^{4k+1}}{(2k)!}}$$