(1) Is $\sum_{n=2}^{\infty} \frac{3}{1-n^2}$ absolutely convergent, conditionally convergent, or divergent?

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Abs Conv?

$$\sum_{n=2}^{\infty} \left|\frac{3}{1-n^2}\right| = \sum_{n=2}^{\infty} \frac{3}{n^2-1}$$

Compare with convergent $\sum_{n=1}^{\infty} \frac{3}{n^2}$ using LCT:

$$\lim_{n\to\infty} \frac{\frac{3}{n^2-1}}{\frac{3}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2-1} = 1 < \infty$$

Thus $\sum_{n=2}^{\infty} \frac{3}{n^2-1} = \sum_{n=2}^{\infty} \left|\frac{3}{1-n^2}\right|$ converges, and $\sum_{n=2}^{\infty} \frac{3}{1-n^2}$ $\boxed{\text{abs conv.}}$

(2) Is $\sum_{k=1}^{\infty} \frac{\cos^5 k}{k^4}$ abs. conv., cond. conv., or div.?

Abs Conv?

Consider $\sum_{k=1}^{\infty} \left| \frac{\cos^5 k}{k^4} \right| = \sum_{k=1}^{\infty} \frac{|\cos^5 k|}{k^4}$.

$$\frac{|\cos^5 k|}{k^4} \leq \frac{1}{k^4}$$

Since $\sum_{k=1}^{\infty} \frac{1}{k^4}$ converges, the smaller $\sum_{k=1}^{\infty} \left| \frac{\cos^5 k}{k^4} \right|$ also converges. Thus $\sum_{k=1}^{\infty} \frac{\cos^5 k}{k^4}$ $\boxed{\text{abs conv}}$.

③ Is $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{4}{n^2+3}$ abs conv, cond conv, or div?

Abs Conv?

$$\sum_{n=0}^{\infty} \left| \cancel{(-1)^{n+1}} \frac{4}{n^2+3} \right| = \sum_{n=0}^{\infty} \frac{4}{n^2+3}$$

(Direct Comp Test)

$$\frac{4}{n^2+3} \leq \frac{4}{n^2}$$

Since $\sum_{n=1}^{\infty} \frac{4}{n^2}$ converges, the smaller $\sum_{n=1}^{\infty} \frac{4}{n^2+3}$ also converges.

Thus $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{4}{n^2+3}$ $\boxed{\text{abs conv}}$.

④ Is $\sum_{i=6}^{\infty} (-1)^i \frac{i}{\sqrt{i^3-7}}$ abs. conv, cond. conv, or div.?

**Abs Conv?**

$$\sum_{i=6}^{\infty} \left| \cancel{(-1)^i} \frac{i}{\sqrt{i^3-7}} \right| = \sum_{i=6}^{\infty} \frac{i}{\sqrt{i^3-7}}$$

Compare with divergent $\sum_{i=6}^{\infty} \frac{i}{i^{3/2}} = \sum_{i=6}^{\infty} \frac{1}{i^{1/2}}$:

$$\lim_{i\to\infty} \frac{\frac{i}{\sqrt{i^3-7}}}{\frac{1}{i^{1/2}}} = \lim_{i\to\infty} \frac{i^{3/2}}{\sqrt{i^3-7}} = \sqrt{\lim_{i\to\infty} \frac{i^3}{i^3-7}} = \sqrt{1} = 1 > 0$$

Thus $\sum_{i=6}^{\infty} \frac{i}{\sqrt{i^3-7}}$ also diverges, and $\sum_{i=6}^{\infty} (-1)^i \frac{i}{\sqrt{i^3-7}}$ does not abs. conv.

**Cond Conv?**

$$\sum_{i=6}^{\infty} (-1)^i \underbrace{\frac{i}{\sqrt{i^3-7}}}_{\text{Positive and non-increasing}}$$

Since $\lim_{i\to\infty} \frac{i}{\sqrt{i^3-7}} = \lim_{i\to\infty} \frac{1}{\sqrt{i - 7/i^2}} = 0$, the series converges (by the Alt. Series Test).

Thus the series is $\boxed{\text{Cond. Conv}}$.

(5) Is $\sum_{n=2}^{\infty} \left(-\frac{3}{5}\right)^n$ abs conv, cond conv, or div?

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Abs Conv?

$$\sum_{n=2}^{\infty} \left|\left(-\frac{3}{5}\right)^n\right| = \sum_{n=2}^{\infty} (1)\left(\frac{3}{5}\right)^n$$

$|r| = |3/5| < 1$

The absolute value series is a convergent geometric series.

Thus the series $\boxed{\text{abs. conv.}}$

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(6) Is $\sum_{n=2}^{\infty} \left(-\frac{5}{3}\right)^n$ abs conv, cond conv, or div?

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Div?

$$\sum_{n=2}^{\infty} (1)\left(-\frac{5}{3}\right)^n$$

$|r| = |-5/3| = 5/3 \geq 1$

The series is a $\boxed{\text{divergent}}$ geometric series.

⑦ Is $\sum_{n=13}^{\infty} (-1)^n \frac{1}{n \ln n}$ abs conv, cond conv, or div?

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Abs conv?

$$\sum_{n=13}^{\infty} \left| \cancel{(-1)^n} \frac{1}{n \ln n} \right| = \sum_{n=13}^{\infty} \frac{1}{n \ln n}$$

Integral Test:

$$\int_{13}^{\infty} \frac{1}{x \ln x} dx = \lim_{b \to \infty} \int_{13}^{b} \frac{1}{x \ln x} dx$$

$$= \lim_{b \to \infty} \int_{\ln 13}^{\ln b} \frac{1}{u} du$$

$$= \left( \lim_{b \to \infty} \ln(\ln b) \right) - \ln(\ln 13) \quad \text{diverges.}$$

Thus the series cannot abs conv.

Cond conv?

$$\sum_{n=13}^{\infty} (-1)^n \frac{1}{n \ln n}$$

positive, nonincreasing

Since $\lim_{n \to \infty} \frac{1}{n \ln n} = 0$, the series converges by AST.

Thus the series is $\boxed{\text{cond. conv.}}$