① Does $\sum_{n=0}^{\infty} \sqrt{\frac{n}{n^4+7}}$ converge or diverge? (Direct Comp.)

$$\sqrt{\frac{n}{n^4+7}} \leq \sqrt{\frac{n}{n^4+0}} = \sqrt{\frac{1}{n^3}} = \frac{1}{n^{3/2}}$$

smaller
denominator

Since $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-Series, the smaller

$\sum_{n=0}^{\infty} \sqrt{\frac{n}{n^4+7}}$ also $\boxed{\text{converges}}$.

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② Does $\sum_{n=3}^{\infty} \frac{4}{n^{0.8}-1}$ converge or diverge? (Direct Comp.)

$$\frac{4}{n^{0.8}-1} \geq \frac{4}{n^{0.8+0}} = \frac{4}{n^{4/5}}$$

bigger
denominator

Since $\sum_{n=3}^{\infty} \frac{4}{n^{4/5}}$ diverges by p-Series, the bigger

$\sum_{n=3}^{\infty} \frac{4}{n^{0.8}-1}$ also $\boxed{\text{diverges}}$.

(3) Does $\sum_{j=2}^{\infty} \frac{e^j}{e^{2j}+1}$ converge or diverge? (Direct Comp)

$$\frac{e^j}{e^{2j}+1} \leq \frac{e^j}{e^{2j}+0} = \left(\frac{e}{e^2}\right)^j = \left(\frac{1}{e}\right)^j$$

smaller denominator

Since $\sum_{j=0}^{\infty} (1)\left(\frac{1}{e}\right)^j$ converges as a geometric series,

the smaller $\sum_{j=2}^{\infty} \frac{e^j}{e^{2j}+1}$ also $\boxed{\text{converges}}$.

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(4) Does $\sum_{k=10}^{\infty} \frac{\sin^2 k}{k^3}$ converge or diverge? (Direct Comp)

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bigger numerator

$$\frac{\sin^2 k}{k^3} \leq \frac{1}{k^3}$$

Since the p-Series $\sum_{k=10}^{\infty} \frac{1}{k^3}$ converges, the smaller $\sum_{k=10}^{\infty} \frac{\sin^2 k}{k^3}$

also $\boxed{\text{Converges}}$.

(5) Does $\sum_{m=4}^{\infty} \frac{1}{\ln m}$ converge or diverge? (Direct Comp.)

$$\frac{1}{\ln m} \geq \frac{1}{m}$$

bigger denominator

Since the Harmonic Series $\sum_{m=1}^{\infty} \frac{1}{m}$ diverges, the bigger $\sum_{m=4}^{\infty} \frac{1}{\ln m}$ also $\boxed{\text{diverges}}$.

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(6) Does $\sum_{n=4}^{\infty} \frac{5}{2n+3}$ converge or diverge? (Direct Comp.)

$\left(\frac{5}{2n+3} \leq \frac{5}{2n} = \frac{5/2}{n} \right.$ doesn't work: inequality is backwards $\Big)$

$$\frac{5}{2n+3} \geq \frac{5}{2n+3n} = \frac{5}{5n} = \frac{1}{n}$$

bigger denominator

Since the Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the bigger $\sum_{n=4}^{\infty} \frac{5}{2n+3}$ also $\boxed{\text{diverges}}$.

7) Does $\sum_{n=0}^{\infty} \sqrt{\frac{n}{n^4+7}}$ converge or diverge? (Limit Comp.)

Similar to $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n^4+0}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ which converges by p-Series.

$$\lim_{n\to\infty} \frac{\sqrt{\frac{n}{n^4+7}}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{n^{1/2}}{\sqrt{n^4+7}} \cdot \frac{n^{3/2}}{1} = \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{\sqrt{n^4}}\sqrt{1+7/n^4}}$$

$$= \frac{1}{\sqrt{1+0}} = 1$$

Since the limit converges, the series $\sum_{n=0}^{\infty} \sqrt{\frac{n}{n^4+7}}$

also $\boxed{\text{converges}}$.

8) Does $\sum_{n=3}^{\infty} \frac{4}{n^{0.8}-1}$ converge or diverge? (Limit comp)

Try comparing with divergent Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$.

$$\lim_{n\to\infty} \frac{\frac{4}{n^{0.8}-1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{4n}{n^{0.8}-1} = \lim_{n\to\infty} \frac{4n^{0.2}}{1-\frac{1}{n^{0.8}}} \text{ diverges to } \infty.$$

Since the limit diverges to infinity, the series $\sum_{n=3}^{\infty} \frac{4}{n^{0.8}-1}$

also $\boxed{\text{diverges.}}$

Alt. Solution for 8

Similar to $\sum \frac{1}{n^{0.8}}$, a divergent p-Series.

$$\lim_{n\to\infty} \frac{\frac{4}{n^{0.8}-1}}{\frac{1}{n^{0.8}}} = \lim_{n\to\infty} \frac{4n^{0.8}}{n^{0.8}-1} = 4 > 0$$

Since the limit is greater than 0, the series

$\sum_{n=3}^{\infty} \frac{4}{n^{0.8}-1}$ also $\boxed{\text{diverges}}$.