① Does $\int_2^\infty \frac{32}{x^3}\,dx$ converge or diverge? If it converges, what is its value?

$$= \lim_{b\to\infty} \int_2^b 32x^{-3}\,dx$$

$$= \lim_{b\to\infty} \left[-16x^{-2}\right]_2^b$$

$$= \left(\lim_{b\to\infty} -\frac{16}{b^2}\right) + \frac{16}{2^2}$$

$$= 0 + \frac{16}{4}$$

$$= \boxed{4}$$

$\boxed{\text{converges}}$

② Does $\int_0^\infty \frac{2y}{y^2+3}\,dy$ converge or diverge? If it converges, what is its value?

$$= \lim_{b\to\infty} \int_0^b \frac{2y}{y^2+3}\,dy$$

Let $u = y^2+3$ $\quad y=b \Rightarrow u=b^2+3$

$du = 2y\,dy$ $\quad y=0 \Rightarrow u=0+3$

$$= \lim_{b\to\infty} \int_3^{b^2+3} \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} \ln(b^2+3) - \ln(3)$$

$\boxed{\text{diverges}}$ (to infinity)

③ Does $\int_e^\infty \frac{1}{\ln(x^x)}\,dx$ converge or diverge? If it converges, what is its value?

$$= \lim_{b\to\infty} \int_e^b \frac{1}{x\ln x}\,dx$$

Let $u = \ln x$ $\quad x = b \Rightarrow u = \ln b$

$du = \frac{1}{x}dx$ $\quad x = e \Rightarrow u = \ln e = 1$

$$= \lim_{b\to\infty} \int_1^{\ln b} \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} \ln(\ln b) - \ln(1)$$

$\boxed{\text{diverges}}$ (to infinity)

---

④ Show that $\int_1^\infty \frac{1}{x^2}\,dx + 1 = \int_0^1 \frac{1}{\sqrt{y}}\,dy$. Then illustrate why.

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty} \left[-x^{-1}\right]_1^b$$

$$= \left(\lim_{b\to\infty} -\frac{1}{b}\right) + \frac{1}{1}$$

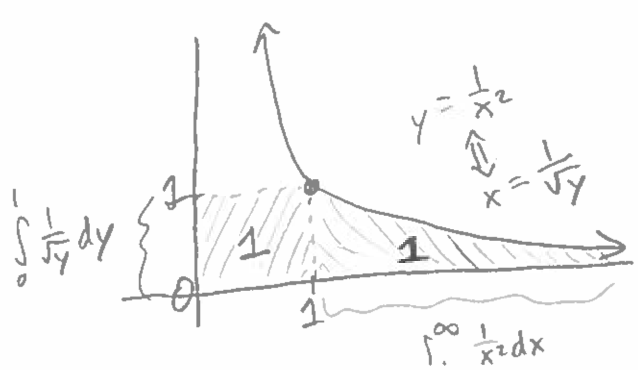
$$= 0 + 1 = 1$$

$$\int_0^1 \frac{1}{\sqrt{y}}\,dy = \lim_{a\to 0^+} \left[2y^{1/2}\right]_a^1$$

$$= 2\sqrt{1} - 2\lim_{a\to 0^+}\sqrt{a}$$

$$= 2 - 0 = 2$$

Thus $\int_1^\infty \frac{1}{x^2}\,dx + 1 = 1 + 1 = 2 = \int_0^1 \frac{1}{\sqrt{y}}\,dy$. □



5. Does $\sum_{n=0}^{\infty} \frac{2n}{n^2+3}$ converge or diverge?

From 2, $\int_0^{\infty} \frac{2x}{x^2+3}\,dx$ diverges, so by the Integral Test, $\sum_{n=0}^{\infty} \frac{2n}{n^2+3}$ also $\boxed{\text{diverges}}$.

---

6. Does $\sum_{n=3}^{\infty} \frac{4}{n(\ln n)^3}$ converge or diverge?

$$\int_3^{\infty} \frac{4}{x(\ln x)^3}\,dx = \lim_{b\to\infty} \int_3^{b} \frac{4}{x(\ln x)^3}\,dx$$

Let $u = \ln x$, $du = \frac{1}{x}\,dx$ $\quad x=b \Rightarrow u=\ln b$, $x=3 \Rightarrow u = \ln 3$

$$= \lim_{b\to\infty} \int_{\ln 3}^{\ln b} \frac{4}{u^3}\,du$$

$$= \lim_{b\to\infty} \left[-\frac{2}{u^2} \right]_{\ln 3}^{\ln b}$$

$$= \left(\lim_{b\to\infty} -\frac{2}{(\ln b)^2} \right) + \frac{2}{(\ln 3)^2}$$

$$= 0 + \frac{2}{(\ln 3)^2}$$

Since the integral converges, the series also $\boxed{\text{converges}}$.