

8) Find an integral for the area bounded by the cardioid $r = 4 + 4\sin\theta$.

$$A = \frac{1}{2} \int_0^{2\pi} (4 + 4\sin\theta)^2 d\theta$$

$$A = \frac{1}{2} \int_0^{2\pi} (16 + 32\sin\theta + 16\sin^2\theta) d\theta$$

$$A = \int_0^{2\pi} (8 + 16\sin\theta + 8\sin^2\theta) d\theta$$

9) Find an integral for the length of the curve $r = \cos^2(\theta)$ where $0 \leq \theta \leq \frac{\pi}{2}$.

$$f(\theta) = \cos^2(\theta)$$

$$f'(\theta) = -2\sin(\theta)\cos(\theta)$$

$$L = \int_0^{\pi/2} \sqrt{(\cos^2\theta)^2 + (-2\sin\theta\cos\theta)^2} d\theta$$

$$= \int_0^{\pi/2} \sqrt{\cos^4\theta + 4\sin^2\theta\cos^2\theta} d\theta$$

$$= \int_0^{\pi/2} \cos\theta \sqrt{\cos^2\theta + 4\sin^2\theta} d\theta$$

$$= \int_0^{\pi/2} \cos\theta \sqrt{1 + 3\sin^2\theta} d\theta$$