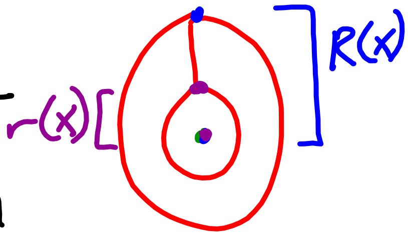
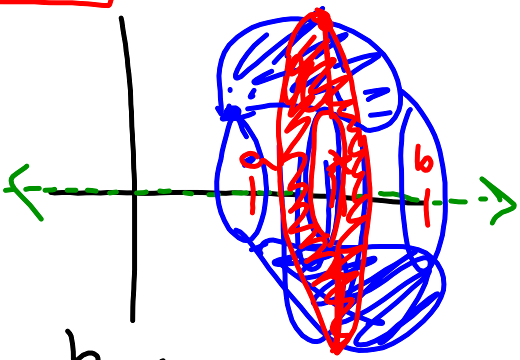
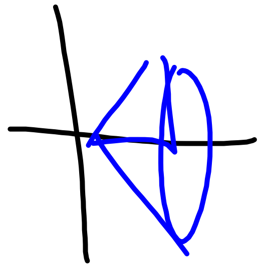
3.3 Washer Method



Solid of Revolution

$$V=\int_a^b \underbrace{A(x)}\,dx$$



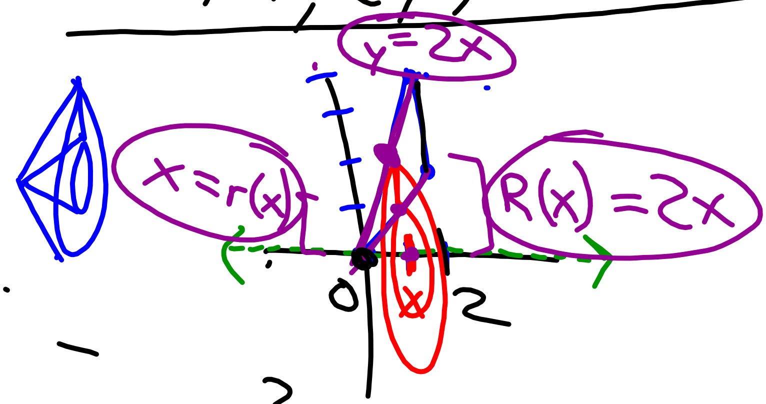


$$V=\int_a^b \left(\pi[R(x)]^2-\pi[r(x)]^2\right)dx$$

$$\boxed{V=\pi\int_a^b\left([R(x)]^2-[r(x)]^2\right)dx}$$

Example

Find the volume of the solid obtained by rotating the triangle with vertices (0,0), (2,2), (2,4) around the x-axis.



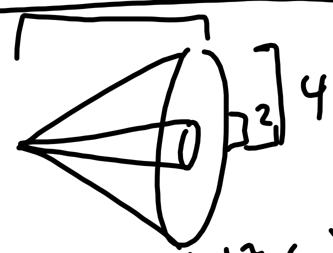
$$V=\pi\int_0^2\left([2x]^2-[x]^2\right)dx$$

$$V=\pi\int_0^2 3x^2\,dx$$

$$=\pi\left[x^3\right]_0^2$$

$$=\pi\left[8-0\right]$$

$$=\boxed{8\pi}$$



$$V=\frac{1}{3}\pi(4)^2(2)$$

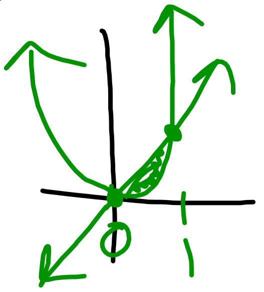
$$-\frac{1}{3}\pi(2)^2(2)$$

$$=\frac{32}{3}\pi-\frac{8}{3}\pi$$

$$=\frac{24}{3}\pi=\boxed{8\pi}$$

Example)

Find the volume of the solid of revolution obtained by rotating the region bounded by $y=x$ and $y=x^2$ around the line $y=2$.



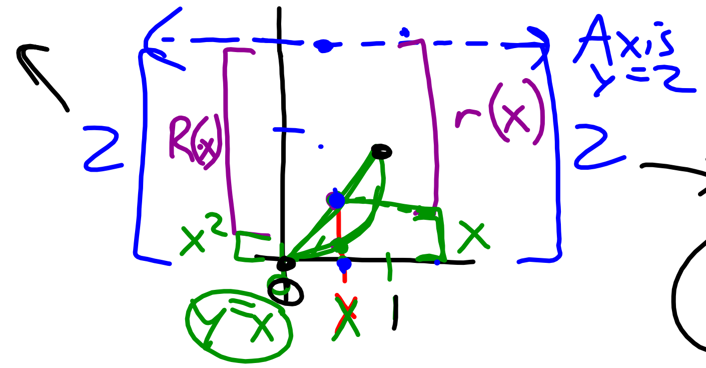
$y=x=x^2$

$0=x^2-x$

$0=x(x-1)$

$R(x)=2-x^2$

$R(x)+x^2=2$



$2=x+r(x)$

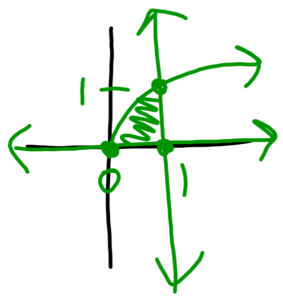
$2-x=r(x)$

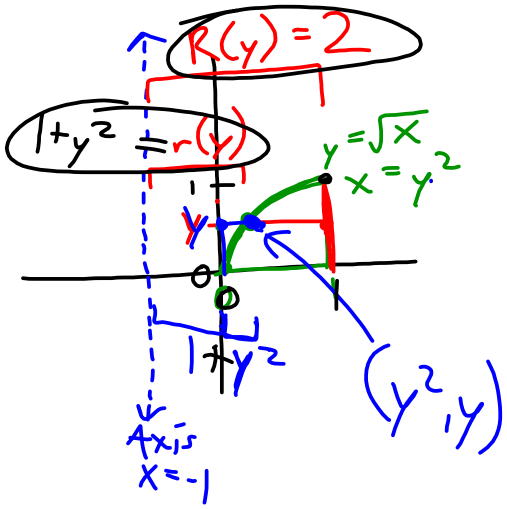
$$V=\pi\int_0^1\left(\left[2-x^2\right]^2-\left[2-x\right]^2\right)dx$$

$$=\ldots$$

$$=$$

Example Find the volume of the solid of revolution obtained by rotating the region bounded by $y=0$, $x=1$, $y=\sqrt{x}$ around the line $x=-1$.





$$V = \pi\int_{y=0}^{y=1}\left([2]^2-[1+y^2]^2\right)dy$$

$$= \pi\int_0^1\left(4-\left(1+2y^2+y^4\right)\right)dy$$

$$= \pi\int_0^1\left(3-2y^2-y^4\right)dy$$

$$= \pi\left[3y-\frac{2}{3}y^3-\frac{1}{5}y^5\right]_0^1$$

$$= \pi\left[\left(3-\frac{2}{3}-\frac{1}{5}\right)-\left(\cancel{0}\right)\right]$$

$$= \frac{\pi(45-10-3)}{15} = \boxed{32\pi/15}$$