① Find $\int (x^2-1)(x^2+1)\,dx$.

(Simplify with algebra first.)

$$= \int \left[x^4 \cancel{- 2x^2 + 2x^2} - 1 \right] dx$$

$$= \boxed{\frac{1}{5}x^5 - x + C}$$

② Find $\int \frac{1}{\sqrt{9+z^2}}\,dz.$

(No u-sub possible; has $a+bx^2 = a + a\tan^2\theta$ form.)

Let $9+z^2 = 9+9\tan^2\theta = 9\sec^2\theta \longrightarrow \sec^2\theta = 1+\frac{1}{9}z^2$

$z^2 = 9\tan^2\theta$

$z = 3\tan\theta \longrightarrow \tan\theta = z/3$

$dz = 3\sec^2\theta\, d\theta$

$$= \int \frac{1}{\cancel{\sqrt{9\sec^2\theta}}} \cancel{3\sec^2\theta}\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \boxed{\ln\left|\sqrt{1+\tfrac{1}{9}z^2} + z/3\right| + C}$$

③ Find $\int 6y^2 e^{y^3}\,dy$.

(y^3 nested in $e^{(\sim)}$, with $y^2 dy$: u-sub)

Let $u = y^3$

$du = 3y^2\,dy$

$2\,du = 6y^2\,dy$

$= \int 2e^u\,du$

$= 2e^u + C$

$= \boxed{2e^{y^3} + C}$

④ Find $\int 3x\sin(4x)\,dx$.

(All techniques fail except Parts.)

Let $u = 3x$ $\qquad v = -\frac{1}{4}\cos(4x)$

$du = 3\,dx \qquad dv = \sin(4x)\,dx$

(To integrate, can use $u = 4x$ if needed)

$$= -\frac{3}{4}x\cos(4x) - \int -\frac{3}{4}\cos(4x)\,dx$$

$$= \boxed{-\frac{3}{4}x\cos(4x) + \frac{3}{16}\sin(4x) + C}$$

5 Find $\int \sec^3\theta \tan^3\theta \, d\theta$.

(Use trig identities to substitute $u = \sec\theta$ or $\tan\theta$.)

$$\cancel{= \int \sec\theta \tan^3\theta \left(\sec^2\theta \, d\theta\right)}$$

$\cancel{\text{Let } u = \tan\theta}$

$\cancel{du = \sec^2\theta \, d\theta}$

← Fails because $\sec\theta$ lacks even power.

$$= \int \sec^2\theta \tan^2\theta \left(\sec\theta \tan\theta \, d\theta\right)$$

Let $u = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$$= \int \sec^2\theta \left(\sec^2\theta - 1\right)\left(\sec\theta \tan\theta \, d\theta\right)$$

$$= \int u^2 \left(u^2 - 1\right) du$$

$$= \int u^4 - u^2 \, du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$= \boxed{\frac{1}{5}\sec^5\theta - \frac{1}{3}\sec^3\theta + C}$$

⑥ Find $\int \frac{5x-5}{x^2-3x-4}\,dx$.

Let $\cancel{u = x^2-3x-4}$
$\cancel{du = (2x-3)\,dx}$
$\cancel{\frac{5}{2}\,du = (5x - \frac{15}{2})\,dx}$,

← Fails because it doesn't match numerator.

(Try partial fractions...)

$$\frac{5x-5}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

$$5x-5 = A(x+1) + B(x-4)$$

Let $x=4$

$$20-5 = A(4+1) + 0$$
$$15 = 5A$$
$$A=3$$

Let $x=-1$

$$-5-5 = 0 + B(-1-4)$$
$$-10 = -5B$$
$$B=2$$

$$= \int \frac{3}{x-4} + \frac{2}{x+1}\,dx = \boxed{3\ln|x-4| + 2\ln|x+1| + C}$$

⑦ Find $\int \frac{3}{2}\sqrt{t} - \frac{1}{t\sqrt{t^2-1}} \, dt$.

$$= \int \underbrace{\tfrac{3}{2} t^{1/2}}_{\left(\text{deriv. of } t^{3/2}\right)} - \underbrace{\frac{1}{t\sqrt{t^2-1}}}_{\left(\text{deriv. of } \sec^{-1}(t)\right)} \, dt$$

$$= \boxed{t^{3/2} - \sec^{-1}(t) + C}$$

8 Find $\int e^x\sqrt{1-e^{2x}}\,dx$.

Let $1-e^{2x} = 1-\sin^2\theta = \cos^2\theta \rightarrow \cos\theta = \sqrt{1-e^{2x}}$

$e^{2x} = \sin^2\theta$

$e^x = \sin\theta \longrightarrow \theta = \sin^{-1}(e^x)$

$e^x\,dx = \cos\theta\,d\theta$

$$= \int \cos\theta\,\cancel{\sqrt{\cos^2\theta}}\,d\theta$$

$$= \int \cos^2\theta\,d\theta$$

$$= \int \frac{1}{2} + \frac{1}{2}\cos 2\theta\,d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$

$$= \boxed{\frac{1}{2}\sin^{-1}(e^x) + \frac{1}{2}e^x\sqrt{1-e^{2x}} + C}$$