5. Find $\int \frac{3x^2+35}{x^3+5x}\,dx$

$$\frac{3x^2+35}{x(x^2+5)} = \frac{A}{x} + \frac{Bx+C}{x^2+5}$$

$$3x^2+35 = A(x^2+5) + (Bx+C)x$$

Let $x=0$

$35 = A(5) + \cancel{0}$

$7 = A$

$$3x^2 + \cancel{35} = 7x^2 + \cancel{35} + Bx^2 + Cx$$
$$-7x^2 - \cancel{35} \quad -7x^2 - \cancel{35}$$

$$-4x^2 = Bx^2 + Cx$$

Coefficients

x^2: $-4 = B$

x: $0 = C$

So $$\int \frac{3x^2+35}{x^3+5x}\,dx = \int \frac{7}{x} - \frac{4x}{x^2+5}\,dx$$

(Let $u = x^2+5$)

$$= \boxed{7\ln|x| - 2\ln|x^2+5| + C}$$

6. Find $\int \frac{2v^3+4v^2+4v+2}{v^2+2v}\,dv$.

(Need long division since num. degree not smaller than denom.)

$$\begin{array}{r} 2v + \frac{4v+2}{v^2+2v} \\ v^2+2v \overline{\smash{\big)}\, \cancel{2v^3} + \cancel{4v^2} + 4v + 2} \\ -(\cancel{2v^3} + \cancel{4v^2}) \\ \hline 4v+2 \end{array}$$

$$\frac{4v+2}{v(v+2)} = \frac{A}{v} + \frac{B}{v+2}$$

$$4v+2 = A(v+2) + Bv$$

Let $v=0$

$2 = A(2)$

$1 = A$

Let $v=-2$

$-8+2 = B(-2)$

$+3 = B$

So

$$= \int 2v + \frac{1}{v} + \frac{3}{v+2}\,dv = \boxed{v^2 + \ln|v| + 3\ln|v+2| + C}$$

⑦ Find $\int \frac{2x^3+6x^2+4x+2}{(x+1)^2(x^2+1)}\,dx$

$$\frac{2x^3+6x^2+4x+2}{(x+1)^2(x^2+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$$

$$2x^3+6x^2+4x+2 = A(x+1)(x^2+1) + B(x^2+1) + (Cx+D)(x+1)^2$$

Let $x=-1$

$\cancel{-2+6-4}+2 = B(2)$

$1 = B$

$$2x^3+6x^2+4x+2 = Ax^3+Ax^2+Ax+A+\cancel{x^2}+\cancel{1}+(Cx+D)(x^2+2x+1)$$

$-x^2 \quad -1 \quad\quad -x^2 \; -1$

$$2x^3+5x^2+4x+1 = Ax^3+Ax^2+Ax+A + Cx^3+2Cx^2+Cx+Dx^2+2Dx+D$$

Coefficients

x^3: $2 = A+C \rightarrow A = 2-C \rightarrow 0 = 2-C \rightarrow C=2$

x^2: $5 = A+2C+D$

x: $4 = A+C+2D \rightarrow 4 = 2-\cancel{C}+C+2D \rightarrow D=1$

Const: $1 = A+D \rightarrow 1 = A+1 \rightarrow A=0$

$$= \int \frac{1}{(x+1)^2} + \frac{2x+1}{x^2+1}\,dx = \int \frac{1}{(x+1)^2} + \frac{2x}{x^2+1} + \frac{1}{x^2+1}\,dx$$

$$= \boxed{-\frac{1}{x+1} + \ln(x^2+1) + \tan^{-1}(x) + C}$$

(8) Describe the expansion of $\frac{f(t)}{(t+1)^2(t^2+9)}$ using partial fractions.

(Assume $f(t)$ is polynomial with degree < 4.)

$$= \boxed{\frac{A}{t+1} + \frac{B}{(t+1)^2} + \frac{Ct+D}{t^2+9}}$$

(9) Find $\int \frac{-x^2+6x-3}{(x+3)(x^2+1)}\,dx$.

$$\frac{-x^2+6x-3}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

$$-x^2+6x-3 = A(x^2+1) + (Bx+C)(x+3)$$

Let $x=-3$

$$-9-18-3 = A(9+1) + 0$$

$$-30 = 10A$$

$$A=-3$$

$$-x^2+6x-\cancel{3} = -\cancel{3x^2}-\cancel{3} + Bx^2+3Bx+Cx+3C$$

$$+3x^2 \quad +3 \quad +3x^2 \quad +3$$

$$2x^2+6x+0 = Bx^2+3Bx+Cx+3C$$

Coefficients

x^2: $2=B$

x: $6=3B+C$
$6=6+0$ ✓

Const: $0=3C$

$0=C$

$$= \int -\frac{3}{x+3} + \frac{2x}{x^2+1}\,dx = \boxed{-3\ln|x+3| + \ln|x^2+1| + C}$$