① Expand $\dfrac{4x^2+16x+17}{(x+2)^3}$.

$$= \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

$$4x^2+16x+17 = A(x+2)^2 + B(x+2) + C$$

$\frac{d}{dx}$

let $x=-2$

$$4(4)+16(-2)+17 = 0 + 0 + C$$

$$1 = C$$

$$8x+16 = 2A(x+2) + B$$

$\frac{d}{dx}$

let $x=-2$

$$-16+16 = 0 + B$$

$$0 = B$$

$$8 = 2A$$

$$4 = A$$

So

$$\frac{4x^2+16x+17}{(x+2)^3} = \boxed{\frac{4}{x+2} \cancel{+ \frac{0}{(x+2)^2}} + \frac{1}{(x+2)^3}}$$

② Expand $\frac{-y^2+2y-4}{(y^2+4)^2}$.

$$= \frac{Ay+B}{y^2+4} + \frac{Cy+D}{(y^2+4)^2}$$

$$-y^2+2y-4 = (Ay+B)(y^2+4) + Cy+D$$

$$0y^3 - 1y^2 + 2y - 4 = Ay^3 + By^2 + 4Ay + 4B + Cy + D$$

Coefficients

y^3: $0 = A$

y^2: $-1 = B$

y: $2 = 4\cancel{A} + C$

$2 = C$

Const: $-4 = 4B + D$

$-4 = -4 + D$

$0 = D$

So

$$\frac{-y^2+2y-4}{(y^2+4)^2} = \boxed{\frac{-1}{y^2+4} + \frac{2y}{(y^2+4)^2}}$$

③ Expand $\dfrac{3r^3+r^2+3}{r^4+3r^2}$.

$$\frac{3r^3+r^2+3}{(r)^2(r^2+3)} = \frac{A}{r} + \frac{B}{r^2} + \frac{Cr+D}{r^2+3}$$

$$3r^3+r^2+3 = Ar(r^2+3) + B(r^2+3) + (Cr+D)(r^2)$$

Let $r=0$

$$3 = 0 + B(3) + 0$$

$$1 = B$$

$$3r^3+\cancel{r^2}+\cancel{3} = Ar^3+3Ar+\cancel{r^2}+\cancel{3}+Cr^3+Dr^2$$

$$\cancel{-r^2}\ \cancel{-3} \qquad\qquad \cancel{-r^2}\ \cancel{-3}$$

$$3r^3 = Ar^3+3Ar+Cr^3+Dr^2$$

Coefficients

r^3: $3 = A+C$ → $C=3$

r^2: $0 = D$

r: $0 = 3A$

$A=0$

So $$\frac{3r^3+r^2+3}{r^4+3r^2} = \cancel{\frac{0}{r}} + \boxed{\frac{1}{r^2}+\frac{3r}{r^2+3}}$$

(4) Find $\int \frac{3z+2}{z^2+2z+1}\,dz$.

$$\frac{3z+2}{(z+1)^2} = \frac{A}{z+1} + \frac{B}{(z+1)^2}$$

$$3z+2 = A(z+1) + B$$

$\frac{d}{dz}$

Let $z=-1$

$3(-1)+2 = \cancel{A(0)}+B$

$-1 = B$

$3 = A$

So

$$\int \frac{3z+2}{z^2+2z+1}\,dz = \int \frac{3}{z+1} - \frac{1}{(z+1)^2}\,dz$$

$$= \int 3\frac{1}{z+1} - (z+1)^{-2}\,dz$$

$$= \boxed{3\ln|z+1| + (z+1)^{-1} + C}$$

$$= \boxed{3\ln|z+1| + \frac{1}{z+1} + C}$$