1. Find $\int 3x \cosh(x)$.

Let $u = 3x$ $\quad v = \sinh(x)$

$du = 3dx$ $\quad dv = \cosh(x)$

$$= \underbrace{3x}_{u}\underbrace{\sinh(x)}_{v} - \int \underbrace{3}_{du}\underbrace{\sinh(x)}_{v}\underbrace{dx}_{du}$$

$$= \boxed{3x\sinh(x) - 3\cosh(x) + C}$$

2. Find $\int te^{2t}\,dt$

Let $u = t$ $\quad v = \frac{1}{2}e^{2t}$

$du = dt$ $\quad dv = e^{2t}dt$

$$= t\left(\frac{1}{2}e^{2t}\right) - \int \frac{1}{2}e^{2t}\,dt$$

$$= \boxed{\frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + C}$$

3 Find $\int y^2 \sin(y)\, dy$.

Let $u = y^2$ $\quad v = -\cos(y)$

$du = 2y\, dy$ $\quad dv = \sin(y)\, dy$

$$= y^2(-\cos y) - \int -\cos(y)(2y\, dy)$$

$$= -y^2 \cos y + \int 2y \cos(y)\, dy$$

Let $u = 2y$ $\quad v = \sin(y)$

$du = 2dy$ $\quad dv = \cos(y)dy$

$$= -y^2 \cos y + \left[2y \sin(y) - \int \sin(y)(2dy)\right]$$

$$= \boxed{-y^2 \cos y + 2y \sin(y) + 2\cos(y) + C}$$

(4) Find

$$\int 4x\sec^2 x\,dx.$$

Let $u = 4x$ $\quad v = \tan x$

$du = 4\,dx$ $\quad dv = \sec^2 x\,dx$

$$= 4x\tan x - \int 4\tan x\,dx$$

$$= \boxed{4x\tan x - 4\ln|\sec x| + C}$$

OR

$$= 4x\tan x - \int 4\tan x\,dx$$

$$= 4x\tan x - 4(-\ln|\cos x|) + C$$

$$= \boxed{4x\tan x + 4\ln|\cos x| + C}$$

(5) Find $\int e^{3w}\sinh(w)\,dw$.

Easy Way

Let $u = e^{3w}$ $\quad v = \cosh(w)$

$du = 3e^{3w}\,dw$ $\quad dv = \sinh(w)\,dw$

$$= e^{3w}\cosh(w) - \int 3e^{3w}\cosh(w)\,dw$$

Let $u = 3e^{3w}$ $\quad v = \sinh(w)$

$du = 9e^{3w}\,dw$ $\quad dv = \cosh(w)\,dw$

$$= e^{3w}\cosh(w) - \left[3e^{3w}\sinh(w) - \int 9e^{3w}\sinh(w)\,dw\right]$$

$$\int e^{3w}\sinh(w)\,dw = e^{3w}\cosh(w) - 3e^{3w}\sinh(w) + \cancel{9\int e^{3w}\sinh(w)\,dw}$$

$$-9\int e^{3w}\sinh(w)\,dw \qquad\qquad \cancel{-9\int e^{3w}\sinh(w)\,dw}$$

$$\frac{-8\int e^{3w}\sinh(w)\,dw}{-8} = \frac{e^{3w}\cosh(w) - 3e^{3w}\sinh(w) + C}{-8}$$

$$\int e^{3w}\sinh(w)\,dw = \boxed{\frac{-e^{3w}\cosh(w) + 3e^{3w}\sinh(w)}{8} + C}$$

Hard Way

Let $u = \sinh(w)$ $\quad v = \frac{1}{3}e^{3w}$

$du = \cosh(w)\,dw$ $\quad dv = e^{3w}\,dw$

$$= \frac{1}{3}e^{3w}\sinh(w) - \int \frac{1}{3}e^{3w}\cosh(w)\,dw$$

Let $u = \cosh(w)$ $\quad v = \frac{1}{9}e^{3w}$

$du = \sinh(w)\,dw$ $\quad dv = \frac{1}{3}e^{3w}\,dw$

$$= \frac{1}{3}e^{3w}\sinh(w) - \left[\frac{1}{9}e^{3w}\cosh(w) - \int \frac{1}{9}e^{3w}\sinh(w)\,dw\right]$$

$$\int e^{3w}\sinh(w)\,dw = \frac{1}{3}e^{3w}\sinh(w) - \frac{1}{9}e^{3w}\cosh(w) + \cancel{\frac{1}{9}\int e^{3w}\sinh(w)\,dw}$$

$$-\frac{1}{9}\int e^{3w}\sinh(w)\,dw \qquad \cancel{-\frac{1}{9}\int e^{3w}\sinh(w)\,dw}$$

$$\frac{8}{9}\int e^{3w}\sinh(w)\,dw = \frac{1}{3}e^{3w}\sinh(w) - \frac{1}{9}e^{3w}\cosh(w) + C$$

$$\int e^{3w}\sinh(w)\,dw = \frac{9}{8}\left(\frac{1}{3}e^{3w}\sinh(w) - \frac{1}{9}e^{3w}\cosh(w)\right) + C$$

$$= \boxed{\frac{3e^{3w}\sinh(w) - e^{3w}\cosh(w)}{8} + C}$$