

① Find $\int 3(3x-5)^3 dx$

Let $u = 3x-5$
 $du = 3 dx$

$= \int (u)^3 du$

$= \frac{1}{4} u^4 + C$

$= \boxed{\frac{1}{4} (3x-5)^4 + C}$

② Find $\int 4e^{r-7} dr$

Let $u = r-7$
 $du = 1 dr$

$= \int 4e^u du$

$= 4e^u + C$

$= \boxed{4e^{r-7} + C}$

③ Find $\int 4v \operatorname{sech}^2(2v^2+1) dv$

Let $u = 2v^2+1$
 $du = 4v dv$

$= \int \operatorname{sech}^2(u) du$

$= \tanh(u) + C$

$= \boxed{\tanh(2v^2+1) + C}$

④ Find $\int \frac{2e^x}{e^x+3} dx$.

Let $u = e^x + 3$
 $du = e^x dx$

$$= \int \frac{2}{u} du$$
$$= 2 \ln|u| + C$$
$$= \boxed{2 \ln(e^x + 3) + C}$$

⑤ Find $\int 2t^3 \sqrt{t^2+1} dt$.

$$= \int t^2 \sqrt{t^2+1} \cdot 2t dt$$

Let $u = t^2 + 1$ $t^2 = u - 1$
 $du = 2t dt$

$$= \int (u-1) \sqrt{u} du$$
$$= \int u^{3/2} - u^{1/2} du$$
$$= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C$$
$$= \boxed{\frac{2}{5} (t^2+1)^{5/2} - \frac{2}{3} (t^2+1)^{3/2} + C}$$

⑥ Find $\int \frac{2(\ln s)^3}{s} ds.$

Let $u = \ln s$
 $du = \frac{1}{s} ds$

$= \int 2u^3 du$

$= 2\left(\frac{1}{4}u^4\right) + C$

$= \boxed{\frac{1}{2}(\ln s)^4 + C}$

⑦ Find $\int \frac{3\sqrt{x}}{2(x^{3/2}+2)^2} dx.$

Let $u = x^{3/2} + 2$
 $du = \frac{3}{2}x^{1/2} dx$

$= \int \frac{du}{(u)^2}$

$= \int u^{-2} du$

$= -|u^{-1} + C$

$= \boxed{-\frac{1}{x^{3/2}+2} + C}$

⑧ Find $\int \frac{\cos(1/y)}{y^2} dy.$

Let $u = 1/y = y^{-1}$
 $du = -y^{-2} dy = -\frac{1}{y^2} dy$
 $-du = +\frac{1}{y^2} dy$

$= -\int \cos(u) du$

$= -\sin(u) + C$

$= \boxed{-\sin(1/y) + C}$