① Show that $f(x) = \cos(x)$ is a solution to the differential equation $f''(x) = -f(x)$, $f'(0) = 0$, $f(0) = 1$.

$f(0) = \cos(0) = \underline{1}$ ✓

$f'(x) = -\sin(x)$

$f'(0) = -\sin(0) = \underline{-0 = 0}$ ✓

$f''(x) = -\left(\cos(x)\right)$

$= -\left(f(x)\right)$ ✓

(2) Show that $f(x) = \sin(3x)$ is a solution to the diff EQ $f''(x) = -9f(x)$, $f'(0) = 3$, $f(0) = 0$.

$f(0) = \sin(3 \cdot 0) = \sin(0) = 0$ ✓

$f'(x) = \cos(3x) \cdot 3 = 3\cos(3x)$

$f'(0) = 3\cos(3 \cdot 0) = 3\cos(0) = 3 \cdot 1 = 3$ ✓

$f''(x) = 3(-\sin(3x) \cdot 3)$

$= -9(\sin(3x))$

$= -9(f(x))$ ✓

③ Find a solution to the Diff EQ
$f''(x) = -f(x)$, $f'(0) = 0$, $f(0) = 4$.

(Same as #1 but with 4 instead of 1.)

Try: $f(x) = 4\cos(x)$

$f(0) = 4\cos(0) = 4(1) = 4$ ✓

$f'(x) = 4(-\sin(x)) = -4\sin(x)$

$f'(0) = -4\sin(0) = -4 \cdot 0 = 0$ ✓

$f''(x) = -(4\cos(x))$

$= -(f(x))$ ✓

(4) Prove that if $\sin x = -\frac{5}{13}$ then $\cos x = \pm\frac{12}{13}$.

$$(\sin x)^2 + (\cos x)^2 = 1$$

$$\left(-\frac{5}{13}\right)^2 + (\cos x)^2 = 1$$

$$\frac{25}{169} + (\cos x)^2 = 1$$

$$(\cos x)^2 = 1 - \frac{25}{169}$$

$$(\cos x)^2 = \frac{144}{169}$$

$$\cos x = \pm\sqrt{\frac{144}{169}}$$

$$\cos x = \pm\frac{12}{13} \qquad \square$$

(5) Find a solution to $f''(x) = -4f(x)$, $f'(0) = 6$, $f(0) = 0$.

(Ideas: $6 = 3 \times 2$, $4 = 2^2$, compare with #2, #3.)

Try $f(x) = 3\sin(2x)$

$f(0) = 3\sin(0) = 0$ ✓

$f'(x) = 6\cos(2x)$

$f'(0) = 6\cos(0) = 6$ ✓

$$f''(x) = 12(-\sin(2x)) = -12\sin(2x)$$
$$= -4(3\sin(2x))$$
$$= -4(f(x)).$$ ✓

⑥ Prove that $-1 \le \sin x \le 1$ and $-1 \le \cos x \le 1$.

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x \le 1$$

$$\sqrt{\sin^2 x} \le \sqrt{1}$$

$$|\sin x| \le 1$$

$$-1 \le \sin x \le 1.$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos^2 x \le 1$$

$$\sqrt{\cos^2 x} \le \sqrt{1}$$

$$|\cos x| \le 1$$

$$-1 \le \cos x \le 1. \quad \square$$