① Prove that $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$.

$$-1 \le \cos\left(\frac{1}{x}\right) \le 1$$

$$-x^2 \le x^2 \cos\left(\frac{1}{x}\right) \le x^2$$

(Squeeze/Sandwich Theorem)

$$\lim_{x \to 0} -x^2 \le \lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) \le \lim_{x \to 0} x^2$$

$$0 \le \lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) \le 0$$

Thus $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$. $\square$

② Prove that $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = 4$.

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \sqrt{4}+2 = 2+2 = 4. \quad \square$$

OR

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \cancel{\frac{4-4}{\sqrt{4}-2}} = \cancel{\frac{0}{0}} \leftarrow \text{indeterminate}$$

(L'Hopital's Rule)

$$\stackrel{\text{L'Hopital's Rule}}{=} \lim_{x\to 4} \frac{d/dx[x-4]}{d/dx[x^{1/2}-2]} = \lim_{x\to 4} \frac{1-0}{\frac{1}{2}x^{-1/2}-0}$$

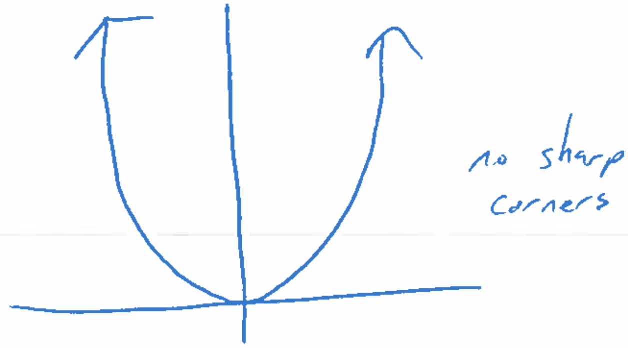
$$= \lim_{x\to 4} 2\sqrt{x} = 2\sqrt{4} = 4. \quad \square$$

③ Give examples of functions defined for all real numbers which are:

1) Differentiable

$f(x) = x^2$

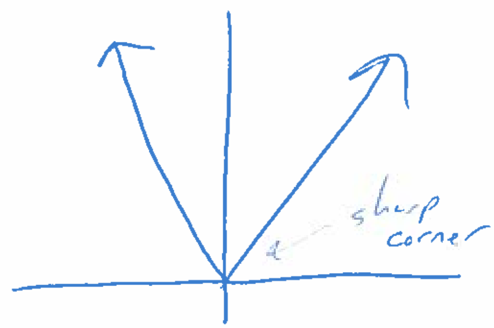
Derivative exists for all x



2) Continuous but not differentiable

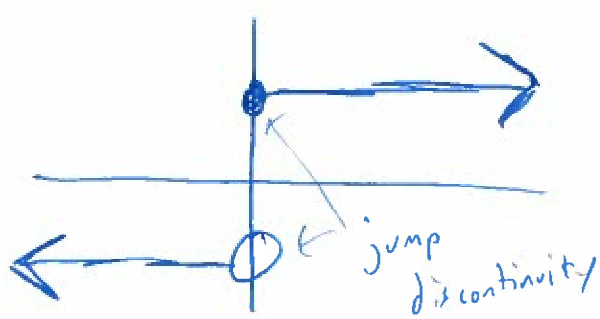
$f(x) = |x|$

Derivative DNE for $x=0$



3) Not continuous

$$f(x) = \begin{cases} 1 : x \geq 0 \\ -1 : x < 0 \end{cases}$$



④ Compute $f'(x)$ for $f(x) = 3 - 5x + 7x^7$.

$$f'(x) = 0 - 5(1) + 7(7x^6)$$
$$= \boxed{-5 + 49x^6}$$

⑤ Compute $f'(x)$ for $f(x) = 3x^2 \tan x$.

$$f(x) = \underbrace{(3x^2)}_{1^{st}}\underbrace{(\tan x)}_{2^{nd}}$$

Product Rule

$$f'(x) = \underbrace{(\tan x)}_{2^{nd}}\underbrace{(6x)}_{D\ 1^{st}} + \underbrace{(3x^2)}_{1^{st}}\underbrace{(\sec^2 x)}_{D\ 2^{nd}}$$
$$= \boxed{6x \tan x + 3x^2 \sec^2 x}$$

⑥ Compute $f'(x)$ for $f(x) = \dfrac{1-x}{4+x^2}$.

$$f'(x) = \frac{(\text{Low})(\text{D High}) - (\text{High})(\text{D Low})}{(\text{Low})^2} \leftarrow \text{Quotient Rule}$$

$$= \frac{(4+x^2)(0-1)-(1-x)(0+2x)}{(4+x^2)^2}$$

$$= \frac{-4-x^2-2x+2x^2}{(4+x^2)^2}$$

$$= \frac{x^2-2x-4}{(4+x^2)^2}$$

⑦ Compute $f'(x)$ for $f(x) = e^{3x+x^3}$.

$$f(x) = \underbrace{e^{(\underbrace{3x+x^3}_{\text{Inside}})}}_{\text{Outside}=e^u}$$

$$f'(x) = \underbrace{e^{(\underbrace{3x+x^3}_{\text{Ins}})}}_{\text{D Out}} \underbrace{(3+3x^2)}_{\text{D Ins}} = \boxed{3e^{3x+x^3} + 3x^2 e^{3x+x^3}}$$

8. Find all antiderivatives of $f(x) = 2x^3 - 5x^4$.

$$\text{All antiderivatives of } f(x) = \int f(x)\,dx$$

$$= \int 2x^3 - 5x^4\,dx$$

$$= 2\left(\tfrac{1}{4}x^4\right) - \cancel{5}\left(\tfrac{1}{\cancel{5}}x^5\right) + C$$

$$= \boxed{\tfrac{1}{2}x^4 - x^5 + C}$$

9. Compute $\frac{d}{dx}\left[\int_1^{x^2} \frac{dt}{1+t^2}\right]$.

(Fund Thm of Calc Part 1: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$)

Inside

$$\frac{d}{dx}\left[\int_1^{x^2} \frac{dt}{1+t^2}\,dt\right] = \frac{1}{1+(x^2)^2}\,(2x)$$

Inside D Inside

(Chain Rule)

$$= \boxed{\frac{2x}{1+x^4}}$$

10 Evaluate $\int_0^{\pi/2} 3\sin x\, dx$.

(Fund Thm of Calc Part 2: $\int_a^b f'(x)\,dx = \left[f(x)\right]_a^b = f(b)-f(a)$.)

$$\int_0^{\pi/2} 3\sin x\, dx = \left[3(-\cos x)\right]_0^{\pi/2}$$

$$= \left(-3\cos\left(\tfrac{\pi}{2}\right)\right) - \left(-3\cos(0)\right)$$

$$= -3(0) + 3(1)$$

$$= \boxed{3}.$$